INFLUENCE OF RAREFACTION ON THE STRUCTURE OF A FREE STREAM OF NITROGEN

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The structure of a free stream behind underexpanded nozzles is studied in relation to geometry of the position of shock waves and boundaries of the stream. A reduction in the pressure level leads to a substantial thickening of the lateral jumps of the compression, and of the Mach disk, and also to an increase of the viscosity effects in the flow core. The experimental data available [1, 2] do not enable us to evaluate the influence of rarefaction on the structure of a free stream. Systematic research, based on the measurement of the density distribution, was therefore undertaken.

By using an electron beam the density distributions on the axis of the stream and in the Mach disk were obtained over a range of Reynolds numbers determined according to parameters in a critical cross section from 188 to 1990, and incalculability from 40 to 2500.

Description of the Equipment and Methods of Measuring Density. Experiments were carried out in an aerodynamic tube of low density [3]. A diagram of the working chamber is given in Fig. 1. The working gas was supplied from the reducer of the vessel 2 through the rheometer 3 into the mixing chamber of the nozzle 4 or through the lateral valve 5 straight into the working chamber. Measurement of pressure in the nozzle mixing chamber is carried out by a U-shaped manometer filled with dibutylphthalate 2. A set of MacLeod manometers of the "Étalon" type 6 is used to measure pressure in the working chamber. The accuracy of the pressure measurements was within 1 to 3%.

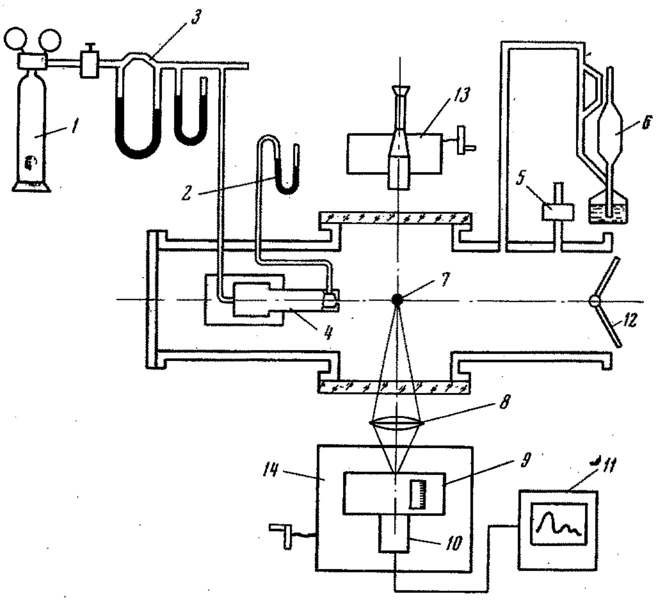

Fig. 1

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An electron gun is fixed in the lower part of the working chamber (perpendicular to the plane of the figure). The beam of electrons with an energy of 17.5 keV and a current of 0.2 mA passes through perpendicularly to the axis of the stream. The mixing chamber 4 together with the nozzle is fixed on a two-component mechanized coordinator with displacements of 150×80 mm^2. By using this coordinator the beam of electrons can be passed through any point of the stream. The part of the luminous column of gas which is excited by the beam of electrons is focused by a quartz lens 8 onto the aperture of the monochromator SPM-2, 9, with a reduction by 1.5 times. The aperture of the monochromator was located perpendicularly to the cluster of electrons. Recording of the luminescence is carried out by the photomultiplier FÉU-39, 10, and recording of the photomultiplier current was carried out by the potentiometer EPPV-60, 11.

Density Measurement. The density measurement was carried out according to the intensity of radiation, excited by a beam of electrons in the visible region of the spectrum [4]. The intensity of the chosen band of nitrogen is given by the expression

$$J = Ch\, c\nu_{jk} A_{jk} n_j \tag{1}$$

Here j is the upper excited state, k is the lower state, h is Planck's constant, ν_{jk} is the transition frequency, A_{jk} is the probability of spontaneous transition, n_j is the number of particles in the upper electron state, c is the speed of light, and C is the instrument constant.

The population of the upper excited state n_j is determined by the expression

$$n_j = i\sigma_{0j}(v)\, n_0 \tag{2}$$

Here i is the current of the electron beam, $\sigma_{0j}(v)$ is the cross section of excitation of the upper state, and n_0 is the number of molecules in the ground state.

Hence accurate measurement of the current of the beam of electrons, and of the radiation intensity, is necessary for density measurement. The usual procedure is to plot a calibration relationship between the radiation intensity and the gas with a known density, and use it for determining the density in unknown conditions.

A number of requirements are listed for the relationship between the suitability for density measurement and the choice of the section of the spectrum, especially:

a) the intensity of the selected lines or strips must be sufficiently high to guarantee a good measuring accuracy;

b) the lifetime of the excited level from which radiation takes place must be small (10^{-7} to 10^{-8} sec). If this condition is not fulfilled it leads to breakdown of radiation in the high-speed flow and an undesirable relationship with the speed.

In order to avoid the influence of resonance absorption, it is desirable to choose a radiation which proceeds from the excited levels of ions (N_2^+, CO_2^+, etc.). With a view to decreasing the influence of secondary electrons the upper excited level must have a high energy. In nitrogen the first negative system of bands satisfies all these requirements. The band (00) of this system was chosen for measurements.

The Order in Which the Experiment Is Conducted. Before the experiment the vacuum system is pumped to a pressure of $\sim 1\,\mu$ Hg. The pressure in the volume of the gun is hence $\sim 10^{-5}$ mm Hg. The preheater of the cathode of the electron gun is then switched on, and it is heated for several minutes. Gas is then supplied through the nozzle into the working chamber, and the presence of the current of the beam on the collector is verified by the microammeter M = 95. By using the coordinator the nozzle is guided right up to the beam, and by lateral displacement of the coordinator of the nozzle unit relative to the fixed electron beam, a point in the flow is found in which the radiation intensity is maximum.

A point situated on the axis of the stream is found in a similar way by vertical displacement of the monochromator 9, which is fixed on a special coordinator 14. After this the nozzle is removed, the gas is supplied through a lateral valve, and calibration of the radiation intensity is carried out using a gas with a known density. The density was determined at room temperature and according to the pressure measurement of the MacLeod manometer. According to the calibration results, the linearity of the relationship between the radiation intensity and the density is maintained up to a pressure of about 90 μ Hg.

TABLE 1

	p_0, mm Hg	$p_k\cdot10^{-3}$, mm Hg	R_*	p_0/p_k	ρ_2/ρ_1	KM	xM/d*
	32.4	12.8	1990	2500	3.52	0.052	33.5
	32.5	20	1994	1630	4.2	0.0412	27.1
1.	32.7	29	2001	1120	4.39	0.0342	22.5
	32.8	37.1	2005	885	4.17	0.0303	19.9
	32.9	50	2010	658	3.84	0.0260	17.2
	28.4	11	1740	2580	4.06	0.0595	34.1
	28.5	17.2	1742	1660	3.16	0.0475	27.3
2.	28.5	25.6	1742	1112	4.37	0.0389	22.4
	28.9	32	1772	903	4.34	0.0345	20.2
	28.8	43	1764	670	3.96	0.0300	17.35
	16.0	7.5	977	2140	2.14	0.096	30.9
	16.0	14.2	977	1125	2.34	0.0697	22.5
3.	16.0	14.6	977	1097	3.06	0.0688	22.2
	16.0	17.6	977	910	3.37	0.0627	20.2
	15.8	27.7	967	573	3.86	0.0500	16.1
	10.75	5.2	657	2070	1.55	0.1405	30.5
	10.9	6.4	670	1704	1.69	0.1265	27.7
4.	10.9	9.5	670	1150	1.97	0.103	22.7
	10.9	12	670	908	2.31	0.092	20.2
	10.9	16.2	670	673	2.64	0.079	17.4
	6.15	3.3	376	1863	1.12	0.234	28.9
5.	6.12	4.6	375	1330	1.20	0.198	24.5
	6.10	6.8	374	898	1.342	0.1635	20.0
	6.10	9.5	374	642	1.50	0.138	16.95
	5.7	12.5	814	456	3.8	0.0533	14.32
	5.85	28.5	835	213	4.35	0.0343	9.78
6.	5.92	33	842	180	4.23	0.0323	8.98
	6.00	51	857	117.8	3.76	0.0258	7.26
	3.82	7.18	545	435	2.47	0.0956	13.95
	3.76	11.1	535	338	2.9	0.0700	12.31
7.	3.69	16.5	530	224	3.49	0.0578	10.04
	3.88	22	553	176	3.96	0.0488	8.9
	3.81	32.5	544	117	3.98	0.0408	7.23
	3.27	8.2	466	390	2.5	0.0883	13.21
	3.28	10	468	328	2.6	0.079	12.11
8.	3.3	14.8	470	223	3.04	0.0647	10.0
	3.3	18	470	183	3.48	0.0588	9.05
	3.3	27.1	470	121.8	3.76	0.0479	7.38
	2.74	7.0	392	392	1.9	0.103	13.25
	2.74	8.1	392	338	2.0	0.0958	12.31
9.	2.76	12.8	393	216	2.61	0.076	9.85
	2.76	15.5	393	178	2.78	0.069	8.95
	2.73	22.5	389	121.6	3.36	0.0575	7.38
	1.965	5.06	280	390	1.47	0.143	13.24
	1.95	5.95	278	320	1.576	0.1333	12.06
10.	1.945	8.9	277	219	1.80	0.1086	9.91
	1.965	10.8	280	182	1.95	0.098	9.04
	1.979	17.6	282	112.3	2.38	0.0765	7.11
	4.74	10	672	474	3	0.0655	14.6
	4.74	16	672	296	3.73	0.052	11.51
	4.74	22	672	215	4.05	0.0442	9.85
11.	4.74	34	672	139	4.16	0.0356	7.9
	4.74	40	672	118	4	0.033	7.26
	4.74	52	672	91	3.8	0.0288	6.4
	4.74	58	672	81.5	3.62	0.0273	6.05
	1.333	4.2	190	317	1.19	0.191	11.9
12.	1.322	6.4	189	206	1.295	0.1552	9.62
	1.301	7.2	186	181	1.34	0.1472	9.00
	1.32	11.1	189	119	1.55	0.118	7.30

After calibration the gas is again supplied through the nozzle. In order to record the density distribution along the axis of the stream, the nozzle unit is moved away to the furthest position, and the electron beam is passed through the stream behind the Mach disk. Recording of the density distribution is carried out from this point with mechanized displacement of the nozzle unit.

The nozzle is stopped at a distance of two to three calibrations from the beam and, by using the optical tube of the cathetometer 13, fixed on the vernier instrument, the final position from the edge of the nozzle is determined. During recording, control of the beam current on the collector is carried out. The recording time is about 15 min.

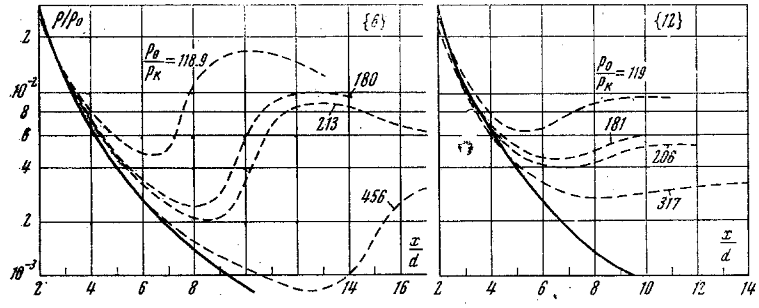

Fig. 2

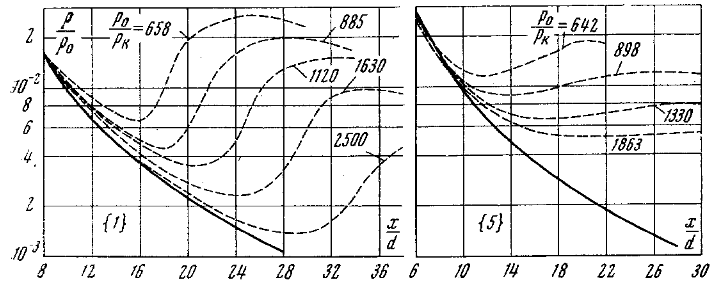

Fig. 3

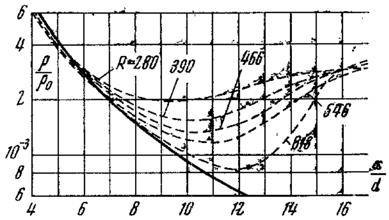

Fig. 4

The relationship between the pressure in the mixing chamber and the pressure in the working chamber (p_0/p_k) is varied by using the flap 12, which throttles the vacuum main pipeline. The range of measured densities is limited by the linear part of the calibration.

Results of Measurements and Their Analysis. The experiments were carried out on sonic nozzles with a critical cross-section diameter of 3 and 7 mm, and a relationship between the thickness of the nozzle wall and the diameter of ~0.02. The conditions of the experiments are given in Table 1. The range of variation of the Reynolds numbers determined according to the parameters of the critical cross section is $R_* = 187$ to 1990. The range of relationships of the pressures in the retarding chamber and in the working chamber p_0/p_k are from 40 to 2500. Table 1 also indicates the distance up to the Mach disk

$$x_M/d^* = 0.67\sqrt{p_0/p_k},\ K_M$$

R is the Knudsen number, determined according to the diameter of the Mach disk $d_M = d^* \times \sqrt{p_0/p_k}$ and according to the length of the free run with a retarding temperature and pressure in the working chamber p_k.

Figures 2 and 3 give some results of determination of the density over the axis of the stream. The experimental conditions are given under the corresponding number in Table 1.

The decrease in the pressure level while maintaining a constant incalculability p_0/p_k (Fig. 4) does not alter the position of the shock wave which is determined according to the point of maximum positive gradient and gives rise only to an increase of the zone between this point and the beginning of withdrawal from the isentropic distribution. A similar result is obtained by Bier [2] in measuring the static pressure.

The departure of the obtained distribution from the isentropic distribution is obviously connected with increase of the zone of formation of the Mach disk, and in the case of small incalculabilities it can be aggravated by the influence of lateral jumps.

The superposition of the expansion process on the formation of a shock wave leads to the fact that the relationship of the density on the Mach disk ρ_2/ρ_1 (ρ_2 is the maximum density and ρ_1 is the minimum density) is not determined by the Hugoniot adiabatic curve, and in the case of a decrease in the pressure level a transition can take place in the shock wave without an increase in the density (Figs. 2 and 3).

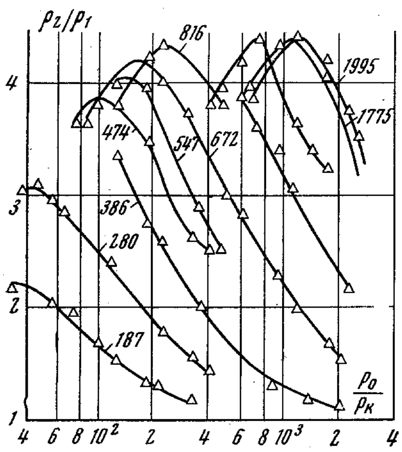

Fig. 5

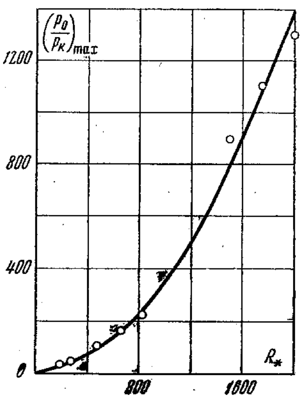

Fig. 6

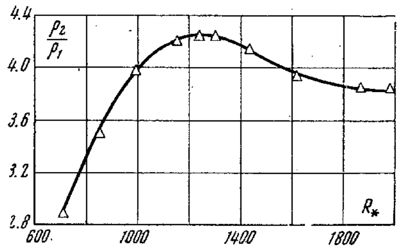

Fig. 7

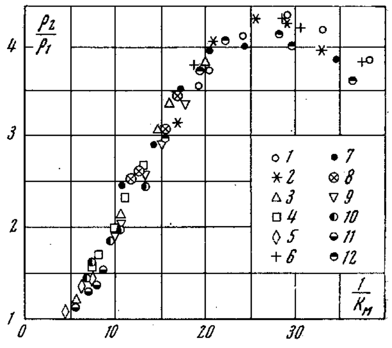

Fig. 8

With a constant R_* the decrease in p_0/p_k leads first of all to an increase in the relationship between the densities ρ_2/ρ_1, and then to a transition through the maximum value, and then a decrease takes place (Fig. 5). The increase in ρ_2/ρ_1 is associated with the fact that the thickness of the shock waves decreases in proportion to $1/p_k$, while the distance up to the Mach disk decreases in proportion to $1/\sqrt{p_k}$. In other words, the decrease in p_0/p_k leads to a decrease in the relative influence of the expansion process. The subsequent decrease of ρ_2/ρ_1 takes place as a result of the fact that the shock wave forms in the region of lower Mach numbers.

The position of the maximum relationship of the densities ρ_2/ρ_1 depends on R_*, and shifts to the side of large values p_0/p_k with increase of R_*. This relationship is satisfactorily described by the relationship $(p_0/p_k)_{max} = AR_*^2$ (Fig. 6, where A is a certain constant).

Figure 7 gives the results of the experiment with a constant p_0/p_k. With increase in R_* the ratio ρ_2/ρ_1 increases basically as a result of the fact that the zone of shock waves contracts, the influence of sphericity decreases, and the formation of a shock wave begins at large Mach numbers. Subsequent decrease in the drop in densities can be associated only with qualitative variation of the flow in the region of the Mach disk. It is possible to make the assumption that for the experiment under consideration, where $R_* = 1200$ in the zone behind the reflected jump of the compression, a supersonic flow begins to form, which has an ejecting action on the central part of the flow (behind the Mach disk). Since this process begins in the rear face of the shock wave, the relative density ρ_2/ρ_1 decreases somewhat.

The analysis carried out shows that the Mach disk in the free low density stream is a zone of shock transition in a viscous expanding flow with possible influence of transverse flows in the zone of the above-mentioned stationary discontinuity.

In the case of constant T_0 and p_0/p_k the criterion R_* reflects in essence the influence of rarefaction on the intensity of the Mach disk. The Knudsen criterion $K_M = \lambda/d_M$, which is determined from the geometrical dimensions of the stream, especially from the diameter of the Mach disk d_M, and the mean length of the free run in the retarded gas behind the Mach disk λ, has close correspondence.

The criterion

$$\frac{\lambda^*}{d^*}\sqrt{p_0/p_k}$$

used by Bier [2] and N. I. Yushchenkova and their colleagues is essentially also a Knudsen number, which differs from that mentioned above by a constant multiple.

Figure 8 represents a generalization of experimental data for ρ_2/ρ_1 according to the parameter K_M. The conditions 1 to 2 correspond with those in Table 1. The right-hand branches of the curves of Fig. 5 are quite satisfactorily confirmed by the generalization.

The investigation reveals a quantitative and qualitative connection between the intensity of the Mach disk in the stream of rarefied gas with the density and the incalculability. In order to build a complete qualitative model of the initial part of the stream behind the sonic nozzle at low density, it is necessary to investigate the conditions of transition from the described viscous flow to one in which the density variation on the shock wave will follow the Hugoniot adiabatic curve.

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